# On $D = 4$, $N = 2$ Supergravity with Abelian electric and magnetic charges

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**Abstract**

We discuss the relation between standard $N = 2$ supergravity with translational gauging and $N = 2$ supergravities with scalar–tensor multiplets with massive tensors and Abelian electric charges. We point out that a symplectic covariant formulation of $N = 2$ supergravity can be achieved just in the presence of tensor multiplets. As a consequence one can see that the formulation of the $N = 2$ theory as it comes from IIB flux compactification, which is included in these models, is equivalent to a non perturbative phase of standard $N = 2$ supergravity. It is also shown that the IIB tadpole cancellation condition is imposed by supersymmetry in four dimensions.

*Key words:* Supergravity, Flux Compactification
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## 1 Introduction

In compactification from IIB supergravity, a double tensor multiplet naturally arises from the zero modes of the RR and NSNS 2–forms and from the complex dilaton [1]. In absence of fluxes the two massless tensors can be dualized into scalars and one obtains the standard $N = 2$ supergravity coupled to vector multiplets and hypermultiplets [2]. In the presence of fluxes the relation with

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standard gauged supergravity becomes less trivial, particularly when magnetic fluxes are included.
In ref. [3], the $N = 2$ gauged supergravity theory coupled to $n_V$ vector multiplets and $n_T$ scalar–tensor multiplets was analyzed, extending a previous work [4] where the $N = 2$ Lagrangian of ungauged supergravity coupled to $n_T$ scalar–tensor multiplet in the absence of vector multiplets was constructed.
Besides the terms due to the semisimple gauging and to the masses of the tensors, as discussed in ref. [3], the fermionic shifts and the scalar potential may also contain terms involving Abelian electric charges [5], [6] which together with the mass terms reconstruct symplectic invariant structures which are the remnant of the IIB electric and magnetic fluxes. The geometrical interpretation of both Abelian electric charges and the mass parameters, becomes more clear in terms of the standard $N = 2$ supergravity where the Abelian isometries, associated to the axions which are dualized into tensors, are gauged before the dualization procedure.
In fact if the ordinary gauging is performed after dualization of the axions into tensors, the reduced scalar manifold does not have any translational symmetry left for the residual quaternionic coordinates, therefore the Abelian electric charges associated to the axionic symmetries cannot appear in the potential. If however the translational gauging is performed before the dualization procedure, then the Abelian electric charges are present from the very beginning in the fermionic shifts, and hence in the potential, as shown in Section 2.
By means of a symplectic rotation, one can generate the mass terms for the tensors , which can be interpreted as magnetic charges [6]. Anyway, such a symplectic rotation acts non perturbatively on the theory, as it is evident on the gauged supergravity side.
For this purpose, in Section 3, we act with a symplectic rotation on the Bianchi identities/equations of motion of the standard gauged supergravity. In this way one gets also a source for the magnetic field strengths and a formulation in terms of a Lagrangian may be problematic. However by a judicious choice of the symplectic transformations, together with the dualization of the axions into tensors, one can still find a Lagrangian.
This dualization procedure clearly shows that the $N = 2$ supergravity which gives a symplectic invariant potential, as it comes from IIB compactifications on a Calabi–Yau in the presence of electric and magnetic fluxes [5,6], must be a theory with scalar–tensor multiplets.
We also show that the supersymmetry Ward identity for the scalar potential, implies:

$$e^I_\Lambda m^{\Lambda J} - e^J_\Lambda m^{\Lambda I} = 0$$

where $e^I_\Lambda, m^{\Lambda J}$ are the electric and magnetic charges, playing the rôle of electric and magnetic fluxes respectively. This corresponds to the ten dimensional tadpole cancellation condition (when we restrict $I = 1, 2$), hence supersymmetry implies that we are in the local case, so that with a *non perturbative* symplectic rotation we can always reduce to the case with only electric charges present.

Let us stress, however, that the Lagrangian is not invariant under non perturbative transformations, therefore such a symplectic rotation is not allowed at the level of the low energy Lagrangian. This means that the Lagrangian which descends from IIB flux compactification, when also magnetic fluxes are present, is not equivalent to the Lagrangian of standard $N = 2$ supergravity with translational gauging. The identification holds just at the perturbative level where the parameters $m^{I\Lambda}$ are zero.

## 2 The dualization with translational gauging

Let us consider the terms in the $N = 2$ bosonic Lagrangian with translational gauging, involving the hypermultiplets scalars and the vector fields [1]:

$$\mathcal{L} = \frac{1}{2}\mathcal{F}^\Lambda\mathcal{G}_\Lambda - h_{uv}dq^u * dq^v - 2h_{Iu}\nabla q^I * dq^u - h_{IJ}\nabla q^I * \nabla q^J - V * \mathbb{1} \quad (1)$$

We have split the quaternionic coordinates in the following way: $q^{\hat{u}} = (q^u, q^I)$, $\hat{u} = 1, \dots 4n$, $I = 1, \dots n_T$, $u = n_T + 1, \dots 4n$, where $n$ is the number of hypermultiplets and $n_T$ is the number of translational isometries on the quaternionic manifold. We chose the coordinates on the quaternionic manifold in such a way that the translations are realized as $q^I \to q^I + \eta^I$, therefore the Lagrangian does not depends on the bare fields $q^I$.

The electric and magnetic field strengths are defined as usual:

$$\mathcal{F}^\Lambda = dA^\Lambda; \quad \mathcal{G}_\Lambda \equiv \frac{\delta\mathcal{L}}{\delta\mathcal{F}^\Lambda} = -Im\mathcal{N}_{\Lambda\Sigma} * \mathcal{F}^\Sigma + Re\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^\Sigma \quad (2)$$

We will focus our attention on the quaternionic manifolds which enjoy the following property:

$$\omega^x_I\omega^x_J = h_{IJ} \quad (3)$$

which certainly holds for Type IIB theory on Calabi–Yau [8], [6], even thougth the whole procedure can be applied to the general case. When equation (3) holds the scalar potential is given by:

$$V = -\frac{1}{2}\mathcal{P}^x_\Lambda(Im\mathcal{N}^{-1})^{\Lambda\Sigma}\mathcal{P}^x_\Sigma; \quad \mathcal{P}^x_\Lambda(q^u) \equiv e^I_\Lambda\omega^x_I(q^u) \quad (4)$$

[1] For all notations and conventions we refer to [3], [7]

where $\mathcal{P}^x_\Lambda$ are the quaternionic prepotentials. The covariant derivative of the axions is defined as:

$$\nabla q^I = dq^I + \mathcal{A}^\Lambda e^I_\Lambda \tag{5}$$

where $e^I_\Lambda$ are constant killing vectors which are $(m+1) \times n_T$ rectangular matrixes which select which combination of the $m+1$ vectors gauges each translational isometry. Since these are $n_T$ linearly independent combinations, by means of a block diagonal symplectic matrix $\mathbf{M}$

$$\mathbf{M} = \begin{pmatrix} \mathcal{M} & 0 \\ 0 & \mathcal{M} \end{pmatrix}; \quad \mathcal{M}^T \mathcal{M} = \mathbb{1} \tag{6}$$

we can go into a basis where each of these combinations coincides with a vector $\mathcal{A}^{\dot\Lambda}$, with $\dot\Lambda = 0 \dots n_T$, in such a way that the rectangular matrix $e^I_\Lambda$ contains as a submatrix the $n_T \times n_T$ matrix $e^I_{\dot\Lambda}$ while all the other entries are zero, that is $e^I_{\tilde\Lambda} = 0$ with $\tilde\Lambda = n_T + 1 \dots m+1$. The equations of motion of the vector fields, together with the Bianchi identities, in this particular basis are given by:

$$d \begin{pmatrix} \mathcal{F}^{\dot\Lambda} \\ \mathcal{F}^{\tilde\Lambda} \\ \mathcal{G}_{\dot\Lambda} \\ \mathcal{G}_{\tilde\Lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2e^I_{\dot\Lambda} \left( h_{Iu} * dq^u + h_{IJ} * \nabla q^J \right) \\ 0 \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ e^I_{\dot\Lambda} J_I \\ 0 \end{pmatrix} \tag{7}$$

Let us consider the equations of motion of the hypermultiplet scalars:

$$\frac{\delta \mathcal{L}}{\delta q^I} = 0 \Rightarrow d(2h_{Iu} * dq^u + 2h_{IJ} * \nabla q^J) = 0 \Leftrightarrow dJ_I = 0 \tag{8}$$

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta q^u} = 0 \Rightarrow \ & d(2h_{uv} * dq^v + 2h_{Iu} * \nabla q^I) - \frac{\partial V}{\partial q^u} + \\ & -\partial_u h_{wv} dq^w * dq^v - 2\partial_u h_{Iv} \nabla q^I * dq^u - \partial_u h_{IJ} \nabla q^I * \nabla q^J = 0 \end{aligned} \tag{9}$$

One can easily see that equation (8) is solved if we set

$$h_{Iu} * dq^u + h_{IJ} * \nabla q^J = dB_I \equiv H_I \tag{10}$$

where $B_I$ is an arbitrary 2–form, such that the equation of motion for the scalars corresponds to the Bianchi identity of the 2–form:

$$d^2 B_I = 0 \tag{11}$$

Applying the Hodge duality to equation (10) and further differentiating, we obtain the equation of motion for the $B_I$'s from the Bianchi identities of the $q^I$'s:

$$d(M^{IJ} * dB_J) - d(A^I_u dq^u) - \mathcal{F}^\Lambda e^I_\Lambda = 0 \tag{12}$$

where we have defined $M^{IJ} h_{JK} = \delta^I_K$ and $h_{Iu} = h_{IJ} A^J_u$. With respect to the ungauged case, the dualization procedure of the scalars $q^I$ gives an electric source term in the equation of motion of the $B_I$'s due to the gauge term in the $q^I$'s Bianchi identity. Considering as the new equations of motion (12) and (9) where (10) is used, we obtain the new Lagrangian:

$$\mathcal{L} = \frac{1}{2}\mathcal{F}^\Lambda \mathcal{G}_\Lambda - 2e^I_\Lambda B_I \mathcal{F}^\Lambda - g_{uv} dq^u * dq^v - 2A^I_u dq^u dB_I - M^{IJ} dB_I * dB_J - V * \mathbb{1} \tag{13}$$

The gauge terms remain unchanged after dualization (16), therefore the scalar potential is the same as in equation (4) and the fermion shifts are given by:

$$\begin{aligned} &\nabla\psi^{(g)}_A = iS_{AB}\gamma_a\psi^B V^a; \quad \nabla\lambda^{iA(g)} = W^{iAB}\psi_B; \quad \nabla\zeta^{(g)}_\alpha = N^A_\alpha \psi_A \\ &S_{AB} = \frac{i}{2}\sigma^x_{AB}\mathcal{P}^x_\Lambda L^\Lambda; \quad W^{iAB} = ig^{i\bar{\jmath}}\sigma^{AB}_x \mathcal{P}^x_\Lambda \bar{f}^\Lambda_{\bar{\jmath}}; \quad N^\alpha_A = -2\mathcal{U}^\alpha_{AI} e^I_\Lambda L^\Lambda \end{aligned} \tag{14}$$

The reduced quaternionic manifold is described, as already found in ref.[4], [3], by the new vielbeins:

$$P^{A\alpha}_u \equiv \mathcal{U}^{A\alpha}_u - A^I_u \mathcal{U}^{A\alpha}_I \tag{15}$$

and the reduced quaternionic connections are related to the old (hatted) ones:

$$\hat\omega^{AB}_u \equiv \omega^{AB}_u + A^I_u \omega^{AB}_I; \; \hat\omega^{AB}_I \equiv \omega^{AB}_I; \; \hat\Delta^{\alpha\beta}_u \equiv \Delta^{\alpha\beta}_u + A^I_u \Delta^{\alpha\beta}_I; \; \hat\Delta^{\alpha\beta}_I \equiv \Delta^{\alpha\beta}_I \tag{16}$$

that is

$$\hat\omega^{AB} = \hat\omega^{AB}_u dq^u + \hat\omega^{AB}_I \nabla q^I = \omega^{AB}_u dq^u - * H^I \omega^{AB}_I \tag{17}$$
$$\mathcal{U}_{\alpha A} = \mathcal{U}_{\alpha A u} dq^u + \mathcal{U}_{\alpha A I}\nabla q^I = P_{\alpha A u} dq^u - \mathcal{U}_{I A\alpha} * H^I \tag{18}$$

Therefore we obtain the following parametrizations for the fermionic fields:

$$D\psi_A = \rho_{Aab} V^a V^b + \epsilon_{AB} T^-_{ab}\gamma^b \psi^B V^a - * H^I_a \omega_{IA}{}^B \psi_B V^a + iS_{AB}\gamma_a \psi^B V^a \tag{19}$$
$$D\lambda^{iA} = D_a\lambda^{iA} V^a + \mathrm{i} Z^i_a \gamma^a \psi^A + G^{-i}_{ab}\gamma^{ab}\psi_B \epsilon^{AB} + W^{iAB}\psi_B \tag{20}$$
$$D\zeta_\alpha = D_a \zeta_\alpha V^a + iP_{aA\alpha}\gamma^a \psi^A - i\mathcal{U}_{IA\alpha} * H^I_a \gamma^a \psi^A + N^A_\alpha \psi_A \tag{21}$$

where the covariant derivatives are defined with the reduced SU(2) and $\mathrm{Sp}(2n,\mathbb{R})$ connections. Since the closure of the Bianchi identities, once the parametrizations of the fermions is given, determines the parametrization of the bosons, the theory we just obtained coincides at the ungauged level with the one constructed in reference [3] [2]. The gauge part differs and describes a "Green–Schwarz" coupling for the electric fields, as the one obtained from IIB compactification in presence of electric fluxes, [8] [6].

## 3 Magnetic fluxes from dualization

If we want to consider also the presence of magnetic fluxes we have to perform a symplectic rotation on the theory. Symplectic transformations [9], [10] are invariances of the Bianchi identities/equations of motion for the ungauged theory but are not in general symmetries of the action as they may mix electric and magnetic field strengths. If the gauging is performed the symmetry between electric and magnetic field strengths does not hold any more due to the presence of a source term for the electric field strengths. This is true also for the dual theory with tensor multiplets as a source term appears just in the vector equation of motion (7).

If one acts with a symplectic rotation on the Bianchi identities/equations of motion on the standard gauged supergravity, one gets also a source for the magnetic field strengths and a perturbative formulation in terms of a Lagrangian may be problematic. Nevertheless, a restriction on the symplectic transformation and the dualizations of the gauged scalar fields into tensors allows us to write a Lagrangian for the new theory.

For this purpose, let us formulate the standard $N = 2$ theory in a symplectic covariant setup. We denote with $< \, , \, >$ the symplectic product

$$< a, b >= \begin{pmatrix} a^\Lambda & b_\Lambda \end{pmatrix} \begin{pmatrix} 0 & \delta_\Lambda{}^\Sigma \\ -\delta^\Lambda{}_\Sigma & 0 \end{pmatrix} \begin{pmatrix} c^\Sigma \\ d_\Sigma \end{pmatrix} = a^\Lambda d_\Lambda - c^\Lambda b_\Lambda \tag{22}$$

and write the gauge covariant derivative of $q^I$ (5) as:

$$\nabla q^I = dq^I + < \mathcal{A}, \mathcal{K}^I >= dq^I + \mathcal{A}^{\dot\Lambda} e^I_{\dot\Lambda} \tag{23}$$

[2] We have corrected a misprinted factor in the parametrization of the gravitino [3].

where we have defined the symplectic vector of electric/magnetic potentials $\mathcal{A}$ and the electric/magnetic killing vector $\mathcal{K}^I$

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}^{\dot{\Lambda}} \\ \mathcal{A}^{\tilde{\Lambda}} \\ \mathcal{A}_{\dot{\Lambda}} \\ \mathcal{A}_{\tilde{\Lambda}} \end{pmatrix}; \qquad \mathcal{K}^I = \begin{pmatrix} 0 \\ 0 \\ e^I_{\dot{\Lambda}} \\ 0 \end{pmatrix} \tag{24}$$

according to:

$$\mathcal{F}^\Lambda = d\mathcal{A}^\Lambda; \qquad \mathcal{G}_\Lambda = d\mathcal{A}_\Lambda \tag{25}$$

Note that we would not be allowed to introduce the magnetic potential $\mathcal{A}_{\dot{\Lambda}}$ for the gauged theory, which is not well defined since $d\mathcal{G}_{\dot{\Lambda}} \neq 0$; nevertheless it does never appear in the Lagrangian and we will manage to never make it appear. We can now perform a symplectic rotation on the theory by means of a matrix $Q$:

$$Q = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{26}$$

where, for the moment, $A$, $B$, $C$, $D$ are taken to be generic matrixes which satisfy the relations which define a symplectic matrix:

$$A^T C = C^T A; \quad B^T D = D^T B; \quad A^T D - C^T B = \mathbb{1} \tag{27}$$

¿From the rotation of the vectors $\mathcal{A}$, $\mathcal{K}^I$ we obtain:

$$\mathcal{A}' = \begin{pmatrix} \mathcal{A}^{\dot{\Lambda}\,\prime} \\ \mathcal{A}^{\tilde{\Lambda}\,\prime} \\ \mathcal{A}'_{\dot{\Lambda}} \\ \mathcal{A}'_{\tilde{\Lambda}} \end{pmatrix} = \begin{pmatrix} A^{\dot{\Lambda}}_{\dot{\Sigma}}\mathcal{A}^{\dot{\Sigma}} + A^{\dot{\Lambda}}_{\tilde{\Sigma}}\mathcal{A}^{\tilde{\Sigma}} + B^{\dot{\Lambda}\dot{\Sigma}}\mathcal{A}_{\dot{\Sigma}} + B^{\dot{\Lambda}\tilde{\Sigma}}\mathcal{A}_{\tilde{\Sigma}} \\ A^{\tilde{\Lambda}}_{\dot{\Sigma}}\mathcal{A}^{\dot{\Sigma}} + A^{\tilde{\Lambda}}_{\tilde{\Sigma}}\mathcal{A}^{\tilde{\Sigma}} + B^{\tilde{\Lambda}\dot{\Sigma}}\mathcal{A}_{\dot{\Sigma}} + B^{\tilde{\Lambda}\tilde{\Sigma}}\mathcal{A}_{\tilde{\Sigma}} \\ C_{\dot{\Lambda}\dot{\Sigma}}\mathcal{A}^{\dot{\Sigma}} + C_{\dot{\Lambda}\tilde{\Sigma}}\mathcal{A}^{\tilde{\Sigma}} + D^{\dot{\Sigma}}_{\dot{\Lambda}}\mathcal{A}_{\dot{\Sigma}} + D^{\tilde{\Sigma}}_{\dot{\Lambda}}\mathcal{A}_{\tilde{\Sigma}} \\ C_{\tilde{\Lambda}\dot{\Sigma}}\mathcal{A}^{\dot{\Sigma}} + C_{\tilde{\Lambda}\tilde{\Sigma}}\mathcal{A}^{\tilde{\Sigma}} + D^{\dot{\Sigma}}_{\tilde{\Lambda}}\mathcal{A}_{\dot{\Sigma}} + D^{\tilde{\Sigma}}_{\tilde{\Lambda}}\mathcal{A}_{\tilde{\Sigma}} \end{pmatrix} \tag{28}$$

$$\mathcal{K}^{I\,\prime} = \begin{pmatrix} m^{I\dot{\Lambda}\,\prime} \\ m^{I\tilde{\Lambda}\,\prime} \\ e^{I\,\prime}_{\dot{\Lambda}} \\ e^{I\,\prime}_{\tilde{\Lambda}} \end{pmatrix} = \begin{pmatrix} B^{\dot{\Lambda}\dot{\Sigma}}e^I_{\dot{\Sigma}} \\ B^{\tilde{\Lambda}\dot{\Sigma}}e^I_{\dot{\Sigma}} \\ D^{\dot{\Sigma}}_{\dot{\Lambda}}e^I_{\dot{\Sigma}} \\ D^{\dot{\Sigma}}_{\tilde{\Lambda}}e^I_{\dot{\Sigma}} \end{pmatrix} \tag{29}$$

If we define the symplectic vector of the new electric and magnetic field strengths as $\mathcal{H}' = d\mathcal{A}'$ the new equations of motion/Bianchi identities are:

$$d\mathcal{H}' = \mathcal{K}^{I\,\prime} J_I \tag{30}$$

The covariant derivative of the scalar becomes:

$$\nabla q^I = dq^I + \mathcal{A}^{\dot{\Lambda}\prime} e^{I\prime}_{\dot{\Lambda}} + \mathcal{A}^{\tilde{\Lambda}\prime} e^{I\prime}_{\tilde{\Lambda}} - \mathcal{A}'_{\dot{\Lambda}} m^{I\dot{\Lambda}\prime} - \mathcal{A}'_{\tilde{\Lambda}} m^{I\tilde{\Lambda}\prime} \tag{31}$$

We easily see from equation (31), (28) that for a generic symplectic rotation the non well defined magnetic potential $\mathcal{A}_{\dot{\Lambda}}$ would appear in the action explicitly, therefore we have to restrict the matrix $Q$ (26) to the transformations which avoid the presence of $\mathcal{A}_{\dot{\Lambda}}$ in the covariant derivative of the scalar fields $\nabla q^I$ (31). This restriction holds also if we want to dualize these scalars into tensors, since as we can see from equation (10), if $\nabla q^I$ is not well defined also the field strength $H_I = dB_I$ would not be well defined.
In order to obtain a sensible but non trivial result, we can restrict the transformation matrix $Q$ to have some vanishing blocks, for instance:

$$B^{\dot{\Lambda}\dot{\Sigma}} = D^{\dot{\Sigma}}_{\tilde{\Lambda}} = 0 \tag{32}$$

This is enough to obtain:

$$m^{I\dot{\Lambda}\prime} = e^{I\prime}_{\tilde{\Lambda}} = 0 \tag{33}$$

and therefore:

$$\mathcal{K}^{I\prime} = \begin{pmatrix} 0 \\ m^{I\tilde{\Lambda}\prime} \\ e^{I\prime}_{\dot{\Lambda}} \\ 0 \end{pmatrix} \;\Rightarrow\; \nabla q^I = dq^I + \mathcal{A}^{\dot{\Lambda}\prime} e^{I\prime}_{\dot{\Lambda}} - \mathcal{A}'_{\tilde{\Lambda}} m^{I\tilde{\Lambda}\prime} \tag{34}$$

Note from equation (28) that thanks to equation (32), the ill defined magnetic potential $\mathcal{A}_{\dot{\Sigma}}$ does not enter the definition of the new potentials $\mathcal{A}^{\dot{\Lambda}\prime}$ and $\mathcal{A}'_{\tilde{\Lambda}}$ which appear in the definition (34). In this case equation(30) becomes:

$$d\begin{pmatrix} \mathcal{F}^{\dot{\Lambda}\prime} \\ \mathcal{G}'_{\dot{\Lambda}} \end{pmatrix} = \begin{pmatrix} 0 \\ e^{I\prime}_{\dot{\Lambda}} J_I \end{pmatrix}; \quad d\begin{pmatrix} \mathcal{F}^{\tilde{\Lambda}\prime} \\ \mathcal{G}'_{\tilde{\Lambda}} \end{pmatrix} = \begin{pmatrix} m^{I\tilde{\Lambda}\prime} J_I \\ 0 \end{pmatrix} \tag{35}$$

Let us now turn to the problem of writing an action which gives equations (35); one immediately realizes that the problem is the presence of the magnetic current, nevertheless we recall that the equation of motion for the fields $q^I$ (8) corresponds to $dJ_I = 0$ which we can solve setting:

$$J_I = 2dB_I \implies dJ_I = 2d^2 B_I = 0 \tag{36}$$

¿From equation (35), using the condition (32) and equation (36) we can read the definitions of $\mathcal{F}^{\dot{\Lambda}\prime}$, $\mathcal{F}^{\tilde{\Lambda}\prime}$, $\mathcal{G}'_{\dot{\Lambda}}$ and $\mathcal{G}'_{\tilde{\Lambda}}$. In fact,defining:

$$\mathcal{C}^{\tilde{\Lambda}} = A^{\dot{\Lambda}}_{\dot{\Sigma}}\mathcal{A}^{\dot{\Sigma}} + A^{\dot{\Lambda}}_{\tilde{\Sigma}}\mathcal{A}^{\tilde{\Sigma}} + B^{\dot{\Lambda}\tilde{\Sigma}}\mathcal{A}_{\tilde{\Sigma}}; \qquad \mathcal{C}_{\tilde{\Lambda}} = \mathcal{A}'_{\tilde{\Lambda}} \tag{37}$$

$$\mathcal{C}_{\dot{\Lambda}} = C_{\dot{\Lambda}\dot{\Sigma}}\mathcal{A}^{\dot{\Sigma}} + C_{\dot{\Lambda}\tilde{\Sigma}}\mathcal{A}^{\tilde{\Sigma}} + D^{\tilde{\Sigma}}_{\dot{\Lambda}}\mathcal{A}_{\tilde{\Sigma}}; \quad \mathcal{C}^{\dot{\Lambda}} = \mathcal{A}^{\dot{\Lambda}\prime} \tag{38}$$

we obtain the expressions for the new electric/magnetic field strengths:

$$\begin{aligned} \mathcal{F}^{\dot{\Lambda}\prime} &= d\mathcal{C}^{\dot{\Lambda}} & \mathcal{F}^{\tilde{\Lambda}\prime} &= d\mathcal{C}^{\tilde{\Lambda}\prime} + 2m^{I\tilde{\Lambda}\prime}B_I \\ \mathcal{G}'_{\dot{\Lambda}} &= d\mathcal{C}_{\dot{\Lambda}} + 2e^{I\prime}_{\dot{\Lambda}}B_I & \mathcal{G}'_{\tilde{\Lambda}} &= d\mathcal{C}_{\tilde{\Lambda}} \end{aligned} \tag{39}$$

which obviously fulfill equations (35), (36). Note, that defining $\mathcal{F}^{\tilde{\Lambda}\prime}$, $\mathcal{G}'_{\dot{\Lambda}}$ in equation (39) we integrated equation (35) using equation (36)

$$d\mathcal{G}_{\dot{\Lambda}} = 2e^I_{\dot{\Lambda}}dB_I \quad \longrightarrow \quad \mathcal{G}_{\dot{\Lambda}} = 2e^I_{\dot{\Lambda}}(B_I + d\Lambda_I) \tag{40}$$

therefore $B_I$ is defined up to an exact 2–form

$$B_I \to B_I + d\Lambda_I \tag{41}$$

where $\Lambda_I$ is a generic 1–form. In order to maintain the invariance of the action one must simultaneously redefine

$$\mathcal{C}^{\tilde{\Lambda}} \to \mathcal{C}^{\tilde{\Lambda}} - 2m^{I\tilde{\Lambda}\prime}\Lambda_I \tag{42}$$

In order to determine completely the Lagrangian which describes the resulting theory, we note that setting $J_I = 2dB_I$ means that we are dualizing the scalars $q^I$ into the tensors $B_I$. Therefore we derive the equation of motion for the fields $B_I$ from the Bianchi identity for the fields $q^I$ as before (10), where now the covariant derivative of the $q^I$'s is given by equation (34) and we obtain the equation of motion for the tensor fields $B_I$:

$$d(M^{IJ} * dB_J) + d(A^I_u dq^u) - (\mathcal{F}^{\Lambda\prime}e^{I\prime}_{\Lambda} - \mathcal{G}'_{\Lambda}m^{I\Lambda\prime}) = 0 \tag{43}$$

As before we can write the dualized Lagrangian, which has the following form:

$$\mathcal{L} = \frac{1}{2}\mathcal{F}^{\Lambda}\mathcal{G}'_{\Lambda} - 2e^{I\prime}_{\Lambda}B_I\mathcal{F}^{\Lambda\prime} - g_{uv}dq^u * dq^v - 2A^I_u dq^u dB_I - M^{IJ}dB_I * dB_J - V' * \mathbb{1} \tag{44}$$

Some comments are in order for the rotated potential $V'$. We can suppose that the symplectic covariant form for the potential $V$ has the following structure:

$$V = -\frac{1}{2}\begin{pmatrix} \mathcal{Q}^{x\Lambda} & \mathcal{P}^x_{\Lambda} \end{pmatrix} \begin{pmatrix} \mathcal{M}_{\Lambda\Sigma} & \mathcal{M}_{\Lambda}^{\ \Sigma} \\ \mathcal{M}^{\Lambda}_{\ \Sigma} & \mathcal{M}^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} \mathcal{Q}^{x\Sigma} \\ \mathcal{P}^x_{\Sigma} \end{pmatrix} \tag{45}$$

where the matrix $\mathcal{M}$ which contracts the vectors of electric/magnetic prepotentials has to be symmetric. We know that

$$\mathcal{M}^{\Lambda\Sigma} = (Im\mathcal{N}^{-1})^{\Lambda\Sigma} \tag{46}$$

since the potential (4) corresponds to the particular case $\mathcal{Q}^{x\Lambda} \equiv m^{I\Lambda}\omega^x_I$. Furthermore as the fermion shifts turn out to be symplectic invariant quantities (49)–(51), thanks to the Ward identity (52) we also know that the scalar potential (45) must be symplectic invariant. Therefore, given one block of such a symmetric symplectic invariant matrix (45) one can determine, with suitable symplectic rotations, all the remaining submatrices. $\mathcal{M}$ turns out to be:

$$\mathcal{M} = \begin{pmatrix} (I+RI^{-1}R)_{\Lambda\Sigma} & -(RI^{-1})_\Lambda{}^\Sigma \\ -(I^{-1}R)^\Lambda{}_\Sigma & (I^{-1})^{\Lambda\Sigma} \end{pmatrix} \tag{47}$$

that is invariant under symplectic transformations, which act both linearly on the matrix and non linearly on the entries (we indicate $I = Im\mathcal{N}$, $R = Re\mathcal{N}$). The transformed potential is therefore given by:

$$V' = -\frac{1}{2} \begin{pmatrix} \mathcal{Q}^{x\Lambda} & \mathcal{P}^x_\Lambda \end{pmatrix} \begin{pmatrix} (I+RI^{-1}R)_{\Lambda\Sigma} & -(RI^{-1})_\Lambda{}^\Sigma \\ -(I^{-1}R)^\Lambda{}_\Sigma & (I^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} \mathcal{Q}^{x\Sigma} \\ \mathcal{P}^x_\Sigma \end{pmatrix} \tag{48}$$

After the symplectic rotation, the fermion shifts (14) are modified and become:

$$\begin{aligned} S'_{AB} &= \frac{i}{2}\sigma^x_{AB}\left(\mathcal{P}^x_\Lambda{}' L^\Lambda{}' - \mathcal{Q}^{x\Lambda}{}' M'_\Lambda\right) = \frac{i}{2}\sigma^x_{AB}\omega^x_I < V, \mathcal{K}^I >' && (49)\\ W^{iAB}{}' &= ig^{i\bar{\jmath}}\sigma^{AB}_x\left(\mathcal{P}^x_\Lambda{}' \bar{f}^{\Lambda}_{\bar{\jmath}}{}' - \mathcal{Q}^{x\Lambda}{}' h'_{\Lambda\bar{\jmath}}\right) = ig^{i\bar{\jmath}}\sigma^{AB}_x\omega^x_I < U_{\bar{\jmath}}, \mathcal{K}^I >' && (50)\\ N^\alpha_A{}' &= -2\mathcal{U}^\alpha_{AI}\left(e^I_\Lambda{}' L^\Lambda{}' - m^{I\Lambda}{}' M'_\Lambda\right) = -2\mathcal{U}^\alpha_{AI} < V, \mathcal{K}^I >' && (51) \end{aligned}$$

where the restriction (33) holds. The supersymmetry ward identity

$$\delta^A_B V = -12 S^{AC} S_{CB} + g_{i\bar{\jmath}} W^{iAC} W^{\bar{\jmath}}_{CB} + 2 N^A_\alpha N^\alpha_B \tag{52}$$

imposes the constraint

$$m^{\Lambda[I} e^{J]}_\Lambda = \frac{1}{2} < \mathcal{K}^I, \mathcal{K}^J > = 0 \tag{53}$$

which in this case it is satisfied thanks to (33).
We also checked the assumption (45) on the structure of the scalar potential is correct, computing explicitly the scalar potential (48) using the fermion shifts (49)–(51) into equation (52).

In the dualization procedure discussed above there are two important points. The first is that the symplectic rotation which one has to perform in order to generate the “magnetic” part of the symplectic product (49)–(51) has to be done with a matrix whith $B \neq 0$, which corresponds to a non perturbative transformation which relates a perturbative gauge theory with coupling constant $g$ with a perturbative theory with coupling constant $1/g$ as $g_{\Lambda\Sigma} = -Im\mathcal{N}_{\Lambda\Sigma}$.
Conversely a supergravity theory with non vanishing $m^{I\Lambda}$ is not equivalent to a standard supergravity with translational gauging, as one has to perform a symplectic transformation which is not an invariance of the action.
The second is that if we perform such a symplectic rotation on the standard gauged supergravity we are forced to dualize the scalars associated to the translational isometries into tensors in order to be able to write an action, which means that the symplectic extension of the standard $N = 2$ theory [7] can be formulated just in terms of scalar–tensor multiplets [11]. Further evidences of this fact can be provided if one imposes the closure of the Bianchi identities as it was done in reference [3], and will be specified to the present case in a forthcoming paper [12].
If we want to make contact with the theory obtained from type IIB compactification on Calabi–Yau in presence of fluxes we consider a double tensor multiplet $(B_I, \tau)$, $I = 1, 2$. The $B_I$'s descend from the RR and the NSNS 2–forms while $\tau = l + ie^{-\phi}$ is the ten dimensional complex dilaton. We find [13] that the connections $\omega^x_I$ are given by:

$$\omega_1^{(1)} = 0;\ \omega_1^{(2)} = 0;\ \omega_1^{(3)} = e^{\varphi};\ \omega_2^{(1)} = -e^{\varphi} Im\tau;\ \omega_2^{(2)} = 0;\ \omega_2^{(3)} = e^{\varphi} Re\tau \quad (54)$$

According to the definition (54) the scalar potential becomes:

$$\begin{aligned} V = &-\frac{1}{2} e^{2\varphi} \left[ \left( e_\Lambda - \overline{\mathcal{N}}_{\Lambda\Sigma} m^\Sigma \right) \left( Im\mathcal{N}^{-1} \right)^{\Lambda\Gamma} \left( \overline{e}_\Gamma - \mathcal{N}_{\Gamma\Delta} \overline{m}^\Delta \right) \right] + \\ &+ 2 Im\tau \left( e_{1\Lambda} m_2^\Lambda - e_{2\Lambda} m_1^\Lambda \right) \end{aligned} \quad (55)$$

where

$$e_\Lambda \equiv e^1_\Lambda + \tau e^2_\Lambda; \qquad m^\Lambda \equiv m^{1\Lambda} + \tau m^{2\Lambda} \quad (56)$$

which coincides with the potential of reference [5] once it is written in the supergravity frame. The tadpole cancellation (53) is now imposed by the supersymmetry Ward identity (52).

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